Uniqueness of the Fock Quantization and Signature Change in cosmology

VI Jerte Advanced Relativity Workshop
9 de abril de 2014

Laura Castelló Gomar
Guillermo A. Mena Marugán
Instituto de Estructura de la Materia, CSIC

Ambiguities in QFT

- The quantization of a classical system is **NOT univocally** defined. Even in linear field theory, one finds **infinitely many** Fock quantizations.

- There exist ambiguities in the choice of:

  - the **Fock representation** of the CCR's

  - the **field description**

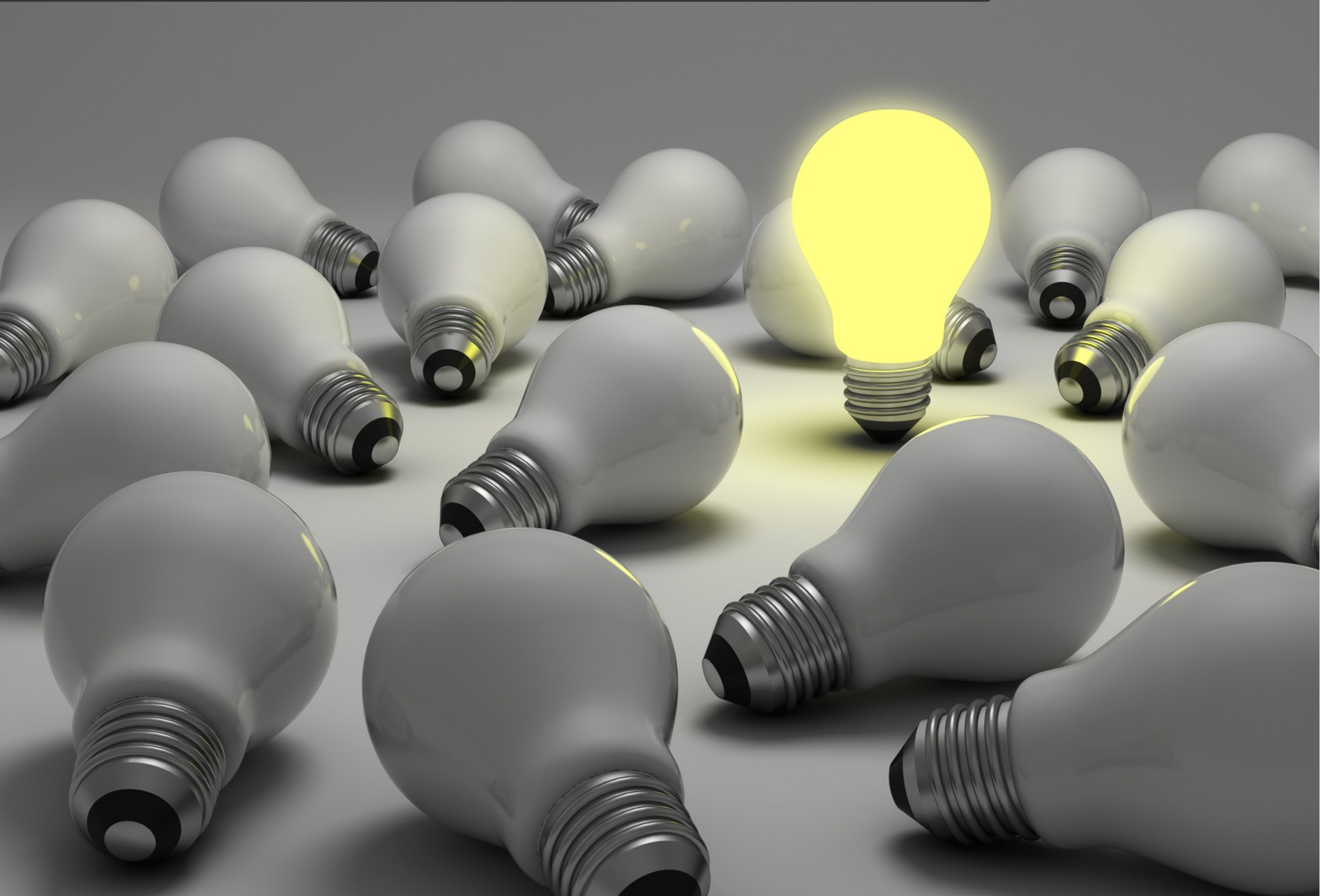
  which are not equivalent.

- In highly symmetric spacetimes, the symmetries of the background can be employed as criteria to determine the quantization (Minkowski spacetime).

- For STATIONARY spacetimes, one can select a quantization with certain requirements on energy.

- In general, systems lack of sufficient symmetry. Recently, **UNIQUENESS** has been reached in some nonstationary scenarios by appealing to the unitarity of the dynamics, rather than to invariance.

Uniqueness criteria



Uniqueness criteria

- Klein-Gordon field in ultrastatic spacetime with time-dependent mass:

$$\varphi'' - \Delta\varphi + m^2(t)\varphi = 0$$

- Our criteria:

  - **INVARIANCE** under the spatial symmetries of the field equations

  - **UNITARY** implementability of the **DYNAMICS** in a finite time interval

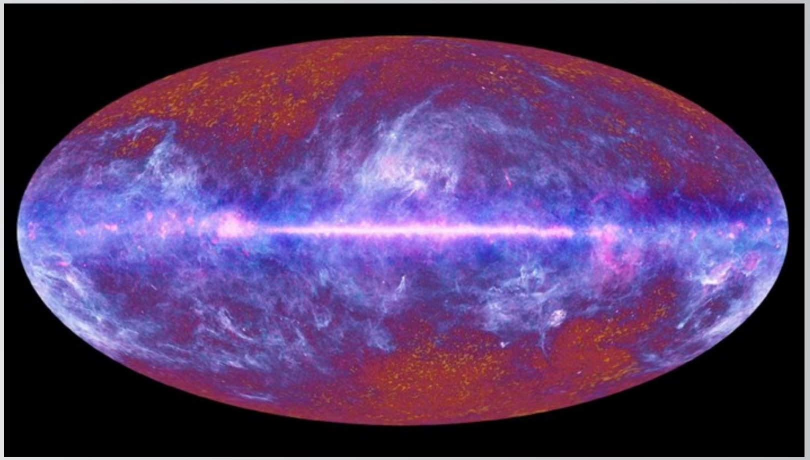
  remove both types of ambiguities, for any (smooth) mass function.

- The uniqueness result is valid for any spatial topology, and at least in any spatial dimension no larger than three.

Motivation: Fields with time dependent mass

RESCALED FIELDS in **FLAT COSMOLOGIES**
(conformal time)

COSMOLOGICAL PERTURBATIONS



- SCALAR PERTURBATIONS:
  Mukhanov-Sasaki variables (gauge invariant).
- PERTURBATIONS of a MASSIVE FIELD in a suitable gauge:
  asymptotic behavior.
- TENSORIAL PERTURBATIONS (gravitational waves).

Motivation: Generalized field equations

- We want to generalize the class of field equations for which we can apply our UNIQUENESS results.

- This would allow us to extend the range of applicability of our criteria.

- In this way, we would cover more general situations in cosmology, obtaining robust quantizations.

- In particular, we would like to study situations with “signature change”. This kind of scenarios have received a lot of attention in LQC recently.

Motivation: Signature change

Motivation: Signature change

- Signature change has already appeared in Quantum Cosmology: think e.g. of the ***tunneling from nothing*** or the **noboundary** proposals.

- Recently, it has reentered the scene in the context of LQC.

- Quantum modifications may lead to a deformed algebra of constraints.

- The corresponding effective equations may behave as if the fields propagated in a background with signature change.

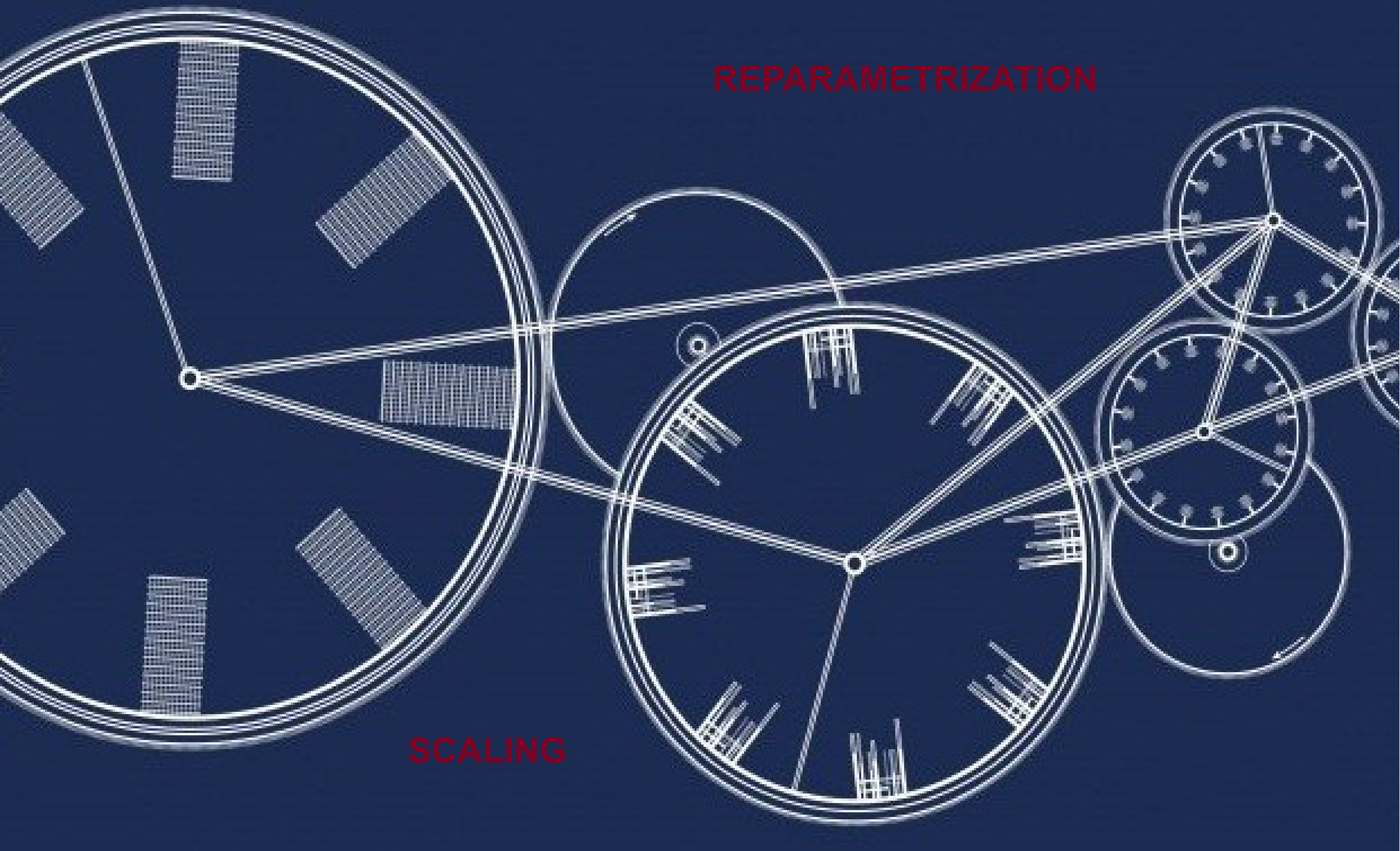
    - Scalar perturbations in a flat FRW background:

$$v'' - \left(1 - \frac{2\rho}{\rho_c}\right) \Delta v - \frac{z''}{z} v = 0$$

Motivation: Signature change

- Can we deal with field equations that involve processes of **signature change**?
- What is the **spacetime interpretation** when these processes are present?
- Can be set **initial conditions** in scenarios with signature change?
- Can this be made compatible with the **uniqueness criteria**?

Generalization of the field equations



Generalization of the field equations

The **most general** second-order equation, maintaining that the spatial variation appears only through the Laplace-Beltrami operator:

$$\phi''+c(t)\phi'-d(t)\Delta\phi+\tilde{m}^2(t)\phi=0$$

$$\phi(t,\vec{x})=f(t)\varphi(t,\vec{x})$$

SCALING

$$dT=g(t)dt$$

REPARAMETRIZATION

$$\varphi''-\Delta\varphi+m^2(t)\varphi=0$$

Unitary implementability is valid for any **time reparametrization**:

$$\hat{U}(t,t_0)\xrightarrow{t=t(T)}\hat{U}(T,T_0)=\hat{U}(t(T),t_0(T_0))$$

Generalization of the field equations

Up to time reversal, there is a **bijective correspondence**:

$$f(t) = C\, d(t)^{-1/4} \exp\left[-\frac{1}{2}\int^{t} c(\bar{t})\, d\bar{t}\right]$$

SCALING

$$g(t) = s\sqrt{d(t)}, \qquad s = \pm$$

REPARAMETRIZATION

We can ALWAYS find an appropriated transformation. Furthermore, it is **UNIQUE**.

Mass:

$$m^2(t) = \frac{\tilde{m}^2(t)}{d(t)} - \frac{d''(t)}{4\mathrm{d}^2(t)} + \frac{5(d'(t))^2}{16\, d^3(t)} - \frac{c'(t)}{2\, d(t)} - \frac{c^2(t)}{4\, d(t)}$$

When the function $d(t)$ vanishes or changes sign.

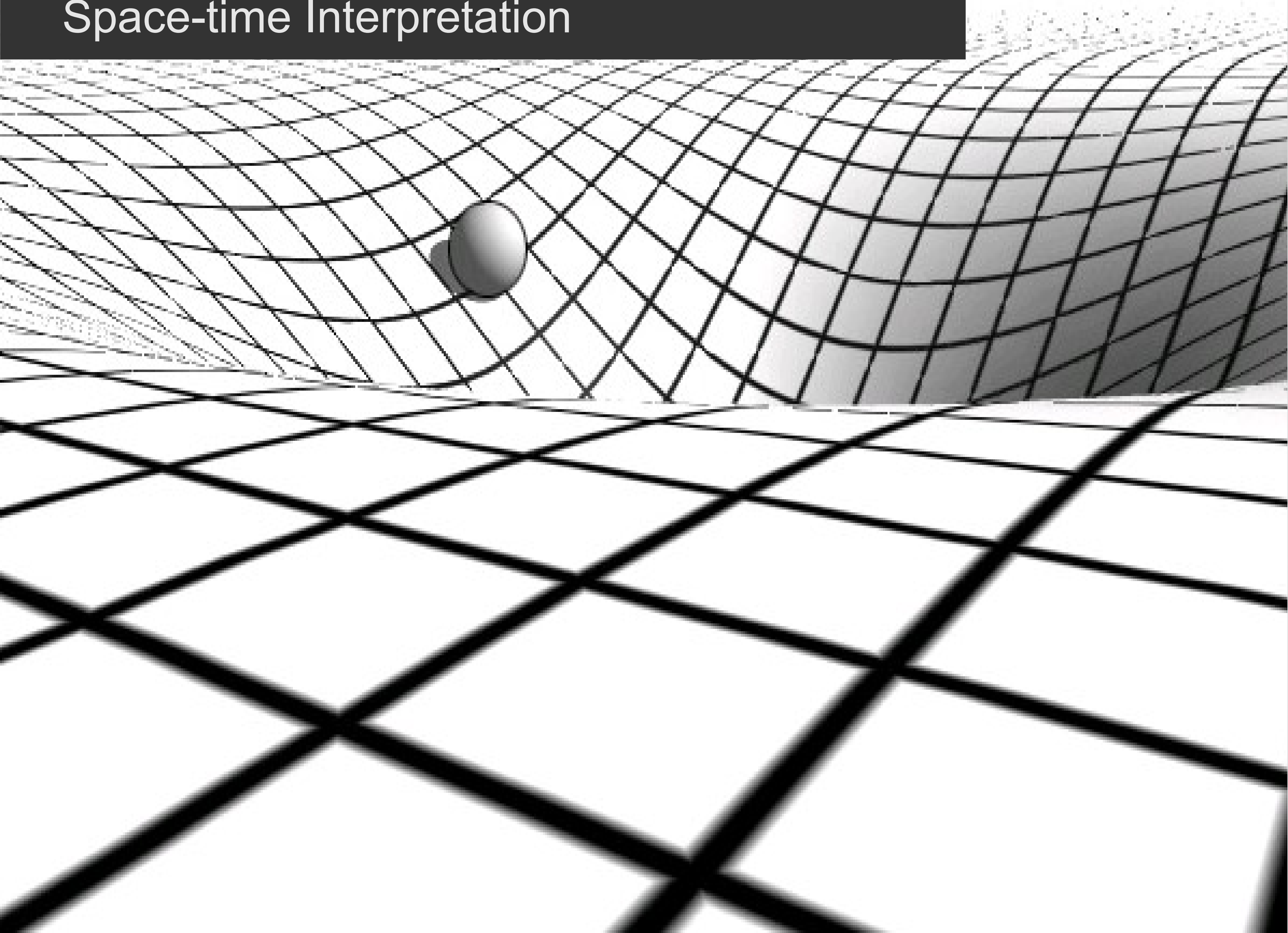
Canonical conjugate **momentums**:

$$P_{\varphi} = f(t)\, P_{\phi} - \sqrt{h}\left(f(t)\, A(t) + \frac{\dot{f}(t)}{f^2(t)}\right)\phi$$

$$P_{\phi} = \sqrt{h}\left(A(t)\,\phi + \frac{1}{f^2(t)}\dot{\phi}\right)$$

- Allowing linear terms in the field.
- Ensuring $P_{\varphi} = \sqrt{h}\,\dot{\phi}$, to apply the criteria of uniqueness.

Space-time Interpretation



Space-time Interpretation

Let us consider a conformally ultrastatic spacetime, with normal spatial sections:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\,h_{ij}(x)\,dx^i\,dx^j$$

$$\downarrow$$

$$\phi'' + \left[\ln\left(\frac{a^3(t)}{N(t)}\right)\right]'\phi' - \frac{N^2(t)}{a^2(t)}\Delta\phi + [N(t)\bar{m}(t)]^2\phi = 0$$

So that:

$$a^4(t) = d(t)\exp\left[\int^t 2c(\bar{t})\,d\bar{t}\right], \qquad N^4(t) = d^3(t)\exp\left[\int^t 2c(\bar{t})\,d\bar{t}\right]$$

The line element is:

$$ds^2 = a^2(t)\left[-d(t)\,dt^2 + h_{ij}(x)\,dx^i\,dx^j\right]$$

Signature change: $(-\ +\ +\ +) \longrightarrow (+\ +\ +\ +)$, if $d<0$.

Space-time Interpretation

- The metric **degenerates completely** when $d(t)$ vanishes.
- If we set $d(t_d)=0$, the metric becomes Euclidean in the region where $d(t)<0$.
- From this perspective, it is more than a signature change. It involves a **singularity** where the scalar curvature explodes as $d^{7/2}$.
- For these geometries, the Ashtekar-Barbero variables behave as:

$$E \sim a^2 \sim \sqrt{|d|} \to 0$$

$$A \sim K \sim d^{-9/4} \to \infty$$

- They become ill defined in the process of signature change.

Vacuum dynamics with signature change

Vacuum dynamics with signature change

We study the evolution of a fixed vacuum state in the **Euclidean** region:

i. We choose a complete set of solutions in the Lorentzian region $\left\{\varphi_n^{\pm}(T)\,\psi_n(\vec{x})\right\}$.

ii. Scaling by the invers of the scale factor and reparametrizating in terms of the time τ corresponding to the lapse $N^2=\epsilon\, a^6,\ \ \epsilon\ =\ \pm,$ we find the set of **solutions** $\left\{\phi_n^{\pm}(\tau)\,\psi_n(\vec{x})\right\}$.

iii. **Wick rotation** of the modes in the Euclidean regime

$$\phi_n^{\pm(E)}\ =\ \lim_{\tilde{\tau}\to i\tau}\phi_n^{\pm}(\tilde{\tau}).$$

iv. The solutions to the field equation $\ddot{\phi}\ =\ -\epsilon\left[a^4\Delta\phi+a^6\bar{m}^2\phi\right]$ can be expressed as a linear combination of these modes with coefficients $c_n^{\pm(E)}$ and $c_n^{\pm}$, respectively, for the Lorentzian and Euclidean regions.

v. We set the initial conditions in τ_0. We require **continuity** conditions of the Klein-Gordon field and its time derivative in the signature change instant, in which the **metric degenerates**.

Vacuum dynamics with signature change

Imposing the continuity conditions, we obtain a linear system for each mode that relates the coefficients of the Euclidean and Lorentzian regions:

$$\begin{pmatrix} c_n^+ \\ c_n^- \end{pmatrix} = \begin{pmatrix} -I_n^{(+-)} & -I_n^{(--)} \\ I_n^{(++)} & I_n^{(-+)} \end{pmatrix} \begin{pmatrix} c_n^{+(E)} \\ c_n^{-(E)} \end{pmatrix}$$

where $$I_n^{(r\,s)} = \lim_{\tau\to 0} \langle \phi_n^{r(E)}(\tau), \phi_n^s(\tau) \rangle, \qquad r, s = + ó -.$$

The field φ with unitary evolution in the lorentzian region:

$$\varphi = a(T) \sum_n \left(c_n^+ \phi_n^+[\tau(T)] + c_n^- \phi_n^-[\tau(T)] \right) \psi_n(\vec{x}).$$

Vacuum dynamics with signature change

If we choose the space defined by the set of positive frequency solutions, $c_n^{+(E)}=0$, the corresponding combination in the lorentziana region have **positive** and **negative** frequencies

$$c_n^{+}=-I_n^{(+-)}, \qquad \boldsymbol{c_n^{-}=I_n^{(++)}}.$$

which leads a **particle production** (exponentially amplified),

Conclusions

- A set of criteria to SELECT a preferred **UNIQUE CLASS** of Fock quantizations for scalar fields in a variety of nonstationary spacetimes with compact spatial topology
- Removing the ambiguities provides physical predictions with great robustness.
- **Generalization** to all the second order equations of motion, through the combination of a scaled field configuration and a time reparametrization, univocally determined.
- **Space-time interpretation** of the considered equation of motion.
- **Signature change** ⟶ elliptic rather than hyperbolic partial differential equations for physical modes.

  ⟶ Space-time **singularity**: there exists a point where the metric is totally degenerated and the scalar invariant curvature becomes infinity.
- Evolution of a vacuum state from a Euclidean to a Lorentzian region.
- Generally, there exists an exponentially amplified **“particle production”**.


NO SIGNAL